*International Journal of Engineering Applied Sciences and Technology, 2023*
**Vol. 8, Issue 08, ISSN No. 2455-2143, Pages 22-28**
**Published Online December 2023 in IJEAST (http://www.ijeast.com)**

# PHYSICS INFORMED NEURAL NETWORK FOR FLOW MODELING AROUND ATHEROSCLEROTIC PLAQUE IN HUMAN CAROTID ARTERY

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*Abstract—* **Unhealthy lifestyle habits have bloomed many life-threatening diseases in humans. Plaque, which causes blood arteries to become narrow or blocked, resulting in atherosclerosis is one of the major causes leading to vascular diseases. The idea of this study is to obtain the characteristics of the blood flow through different contents of plaque without discretization of mathematical model formed by sets of partial differential equation for conservation of mass and momentum. Python-based Physics Informed Neural Network is used in this study to solve the one-dimensional blood flow model for different content of plaque in human carotid artery. Pulsatile boundary condition is provided to the model. It is found that the increase of plaque in the artery wall decreases the overall flow of blood inside the vessel consequently decreasing the flow rate over the vessel with peak velocity in the maximum plaque region. After plaque increases beyond 50% of initial radius (75% of initial area), the rate of flow is found to be insignificant, hence blocking the flow of oxygenated blood to reach the head part of the human body including brain which points towards the medical emergency. For the pressure inside the vessel, it is established that the pressure depends upon the plaques elasticity over its geometry. These outcomes provide additional knowledge about wave propagation in arteries and furthermore endorses the convenience of using neural networks in mathematical flow models.**

*Keywords—* **PINN, Atherosclerosis, Plaque, Carotid artery, Cardiovascular Disease, Volumetric Flow Rate**

## I. INTRODUCTION

Plaque, the fatty substance made up of different biological components such as lipids, foam cell and media, is the major cause of the cardiovascular disease leading to many deaths around the world. Arteries, where plaque forms, are the blood carrying vessels that carry blood from heart to different vital parts of the body. The endothelium part of the artery keeps the arteries wall smooth and within shape, that helps in the flow of blood. Atherosclerosis, arising from the plaque, is the condition where plaque gets deposited in the walls of the artery and obstructs the flow of blood[1]. This obstruction can cause the blockage of the artery and prevents the oxygen from reaching to the vital organs of the body. Studies have found that, Internal Carotid Arteries (ICA) which carry blood to the head part of the human body, is more prone to atherosclerotic plaque. Doppler Ultrasound is often the first imaging to be carried out for arterial disease & with good reason [2]. Different studies related to the plaque has shown the dependence of flow of blood through plaque with temperature, stress and different mechanical behaviors [3]. Different simulations have been presented with the mathematical model consisting of plaques in the artery for flow velocity and blood pressure yield [4].

Computational engineering has been growing in the field of analysis of different engineering practices. Flow physics related to mechanical engineering especially uses the computational techniques required for the design of nanoscale devices and nanostructured materials [5]. The applications of computational engineering range from the motion of molecules along with their transportation to the atomic-scale physics of material behavior. Partial differential equation (PDE) is any mathematical equation that defines the characteristics of the physical phenomena and uses two or more independent variables associated with the system [6]. The mathematical models representing physics are defined using Partial differential equations (PDEs). Partial differential equations (PDEs) provide a significant contribution in the mathematical research and acquires important methods to mimic the physical environment variables into the set of equations. Partial differential equations (PDEs) arise from many conditional assumptions, such as variation of calculus and geometries [7]. Physics informed neural network (PINN), a network of artificial neurons that are capable to make self-decisions as the human brain operates, also embeds the prior knowledge of any physical laws and can also overcome the low data availability of the system [8]. It is found that PINN reduces varieties of cost necessary for biological experiment in micro fluids [9]. Prediction of movement of red blood cells with other possibilities of using neural networks in the field of blood flow informatics is also acclaimed [10].

Generally, the study of mathematical blood flow models governed by PDEs is done using traditional discretization methods such as Finite Difference Method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM) [11]–[14]. As the field of machine learning is forging ahead, better techniques are emerging in the field of computational science. This work uses the method of neural networks directly to the governing equations and the boundary conditions associated with the mathematical models. The implementation of PINN is expected to minimize the truncation errors induced by the previous traditional methods as the discretization of differential equations is not required [15]. The python-based physics informed neural network employed to solve the one-dimensional blood flow models for atherosclerosis could help understand how the blood pressure waves acts upon different content of plaque inside the artery more clearly [16]. A one-dimensional model, obtained by using the conservation of mass and momentum of the fluid with reasonable boundary conditions, is adequate for achieving the required velocity and pressure wave forms [17].

## II. METHODS

### *A.* Artery geometry

In this study, six different incremental plaque contents are used. The geometries differ in terms of content of plaque stacked in the inner walls of the arteries.

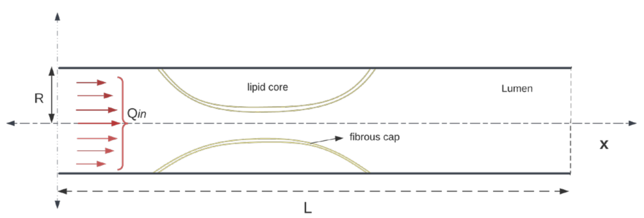


Fig. 1. Geometry of internal carotid artery and plaque

The mathematical formulation used to define the variable geometry with certain percentage decrease of area due to plaque is given as [18],

$$R = \begin{cases} 0.25 * \left(1 - \frac{cp}{2}\left(1 + \cos(5\pi x)\right)\right) & 2 \le x \le 6 \\ 0.25 & elsewhere \end{cases}$$

where, R is the radius of the artery, which is taken as 0.25 cm as literature suggests for internal carotid arteries [19]. cp is the percentage reduction in radius with respect to the radius itself.

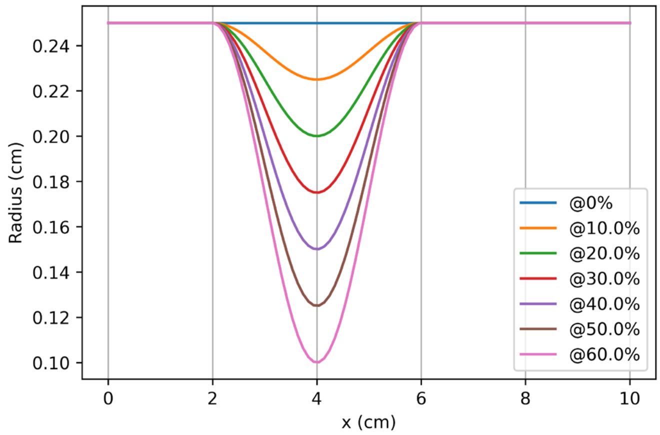


Fig. 2. Radius(R) of artery for different contents of plaque

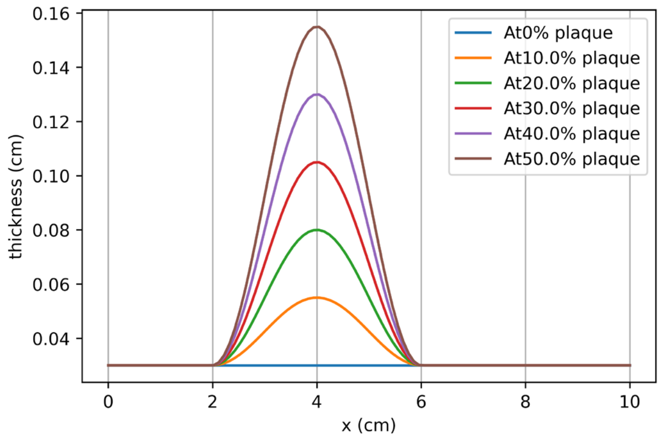


Fig. 3. Thickness($h_0$) of artery for different contents of plaque

The thickness ($h_0$) is also made to vary with different conditions of plaque. The thickness of the plaque free artery is taken as 0.03 cm [20]. The reduction in the radius is compensated by the increase in thickness in the plaque area.

### *B.* One dimensional model

The one dimensional (1D) wave propagation model is used for this study. The use of 1D models with FEM techniques can be seen in most of the literatures where wave analysis of pressure and flow rate is achieved [21]. 3D models are computationally expensive, use of 1D model is sufficient

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enough to obtain the information about propagative phenomena such as velocity and pressure [17]. Their low cost of computation makes it possible to study the global as well as local circulation system of arteries around the body [22]. However, one dimensional models lack in providing the flow-field related details sufficient to permit calculations of local quantities such as re-circulation of blood and oscillating shear stresses to ta walls of blood vessel [23]. One dimensional blood flow model is derived from the Navier Stokes equations.

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \frac{1}{\rho}\nabla P - div[v(\nabla u + (\nabla u)^T)] = 0$$

$$div\, u = 0$$

Applicable on the cylindrical vessel of the artery that changes in time to accompany movement of wall due to flow. The main assumptions behind the derivation to one dimensional blood flow model are,

i. The domain is cylindrical at all times with axial vessel length being time-invariant.

ii. The viscous effects are significant near the walls only.

iii. The artery wall is assumed to be impermeable (leak proof).

By taking the assumptions into account, we obtain the following system of two partial differential equation (for full derivation, see [24])

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial t}\left(\alpha^2 \frac{Q}{A}\right) + \frac{A}{\rho}\frac{\partial P}{\partial x} + K_R \frac{Q}{A} = 0$$

The sets of equation above represents the conservation of mass and momentum where x is the axial direction, A is the area of the cross-section, ρ is the density of the blood which is taken to be constant, Q represents the averaged volumetric flow rate across the section.

α = 1, is used for this study assuming the flat velocity profile while for a parabolic profile, α = 4/3 is also suggested furthermore, this assumption simplifies the model to some extent.

As for the resistance parameter ($K_R$), it represents the viscous resistance of the flow per unit area defined by $K_R = 8\pi v$ [17], where v represents the kinematic viscosity of the blood.

The unknowns in the system of two partial differential equations are Q, P, A in which the number of unknown exceeds the number of equations. To complete the system explicitly, an algebraic relation between the pressure P and the vessel area A is provided given from the literature [23]as,

$$P = P_{ext} + \beta\left(\sqrt{A} - \sqrt{A_0}\right)$$

where,

$$\beta = \frac{E h_0 \sqrt{\pi}}{(1-\gamma^2)A_0}$$

here, β represents the mechanical behaviour of the vessel, $A_0$ represents the sectional area and $h_0$ represents the vessel thickness at equilibrium state. E represents the Young modulus of the vessel material and $P_{ext}$ is the external pressure, assumed zero, and ϒ is Poisson ratio which is taken as ϒ = 0.5, since biological artery tissue is practically in-compressible [17].

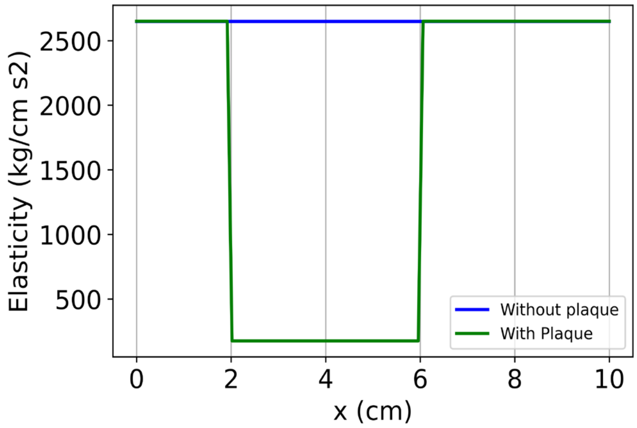


Fig. 4. Elastic modulus for the artery

As the mechanical behaviour, β primarily depends upon the elastic modulus E, there is a difference of value of elastic modulus for vessel material and plaque material. The vessel's elastic modulus is taken as 2650 kg/cm $s^2$ and the value for plaque is much less at 176 kg/cm $s^2$ as the core of the plaque is made up of different lipids [25].

The constant parameters to be used in the mathematical model can be found in Table-1.

Table -1 Blood Flow constants used in the model

| **Property** | **Value** |
|---|---|
| Density (ρ) | 0.00106 $kg/cm^3$ |
| Kinematic Viscosity (ν) | 0.035 $cm^2/s$ |
| Coriolis Coefficient (α) | 1 |
| Elastic modulus (E) (plaque) | 176 $kg/cm.s^2$ |
| Elastic modulus (E) (artery wall) | 2650 $kg/cm.s^2$ |

*C.* **Boundary Conditions**

The characteristic analysis of mass and momentum conservation shows that the two equation system is strictly hyperbolic for A>0 and hence contains two real eigenvalues having opposite sign to each other. So, the differential PDE requires one boundary condition at x = 0 and one boundary condition with input characteristics at x = L [26]. The end of the vessel is expected to have a non-reflecting boundary condition where the input waves are not reflected after hitting the outlet condition. Boundary conditions could be expressed directly as pertinent characteristics variables used in the mathematical equations whenever explicit formulation of the variables is available, but the available data is usually in terms of the known physical variable associated with the governing equation. If the explicit expression for the required characteristic variable is unavailable, pseudo-characteristic approach can also be adopted instead. So, the input pressure wave is provided at the initial stage of the vessel along with the constant flow rate of 5 ml/s.

$$P_{in} = \begin{cases} 133.32 * (1-k)\sin(\pi t) & 0 \le t \le 0.5 \\ 133.32 * (1-k\cos(2\pi(t-0.5))) & t > 0.5 \end{cases}$$

where k is the relative amplitude taken as 1/4 for this study.

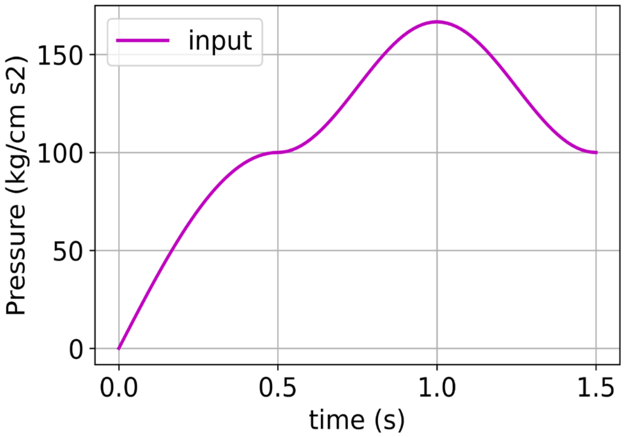


Fig. 5. Input Pressure wave

For this study, the initial condition constitutes of no pressure i.e. P(x,0) = 0 so that, A(x,t=0) = $A_0$. In the inlet condition, input wave pressure is provided as the input. The wave fluctuates around the mean position of 133.33 kg/cm $s^2$ i.e. 100 mm hg of pressure as shown in Fig 5. The output condition consists of the minimal pressure exerted by the veins considered as constant. Also, the input of volumetric flow at initial condition is provided for the blood to flow through the artery domain.

### *D.* Physics Informed Neural Network

A neural network (NN) is a circuit of connections of neurons that mimics the way the human brain operates. NN basically is the set of codes of computer programs that perform complex optimization problems by the process of iterative training on the known data set. The trained model acquires the highest possible accuracy using various tools and techniques like different types of optimization functions, activation functions, loss functions, different encoding methods and back-propagation. There are user-defined layers between the input and output of the model. Each layer contains further user-defined neurons well connected which are excited by the activation functions. Physics Informed Neural Network (PINN) is the advanced form of artificial neural network that is instructed to solve learning tasks while considering given laws of physics described by general non-linear partial differential equations [8]. In PINNs, automatic differentiation is embedded to evaluate differentiation without discretization, that help in reducing the errors and multi-task training with learning to promote high computational abilities [6]. The idea of PINNs is to learn from the differences between target and input values associated with the laws of physics and scientific knowledge. This enables PINN into learning algorithms that make them more robust and improve their overall performance.

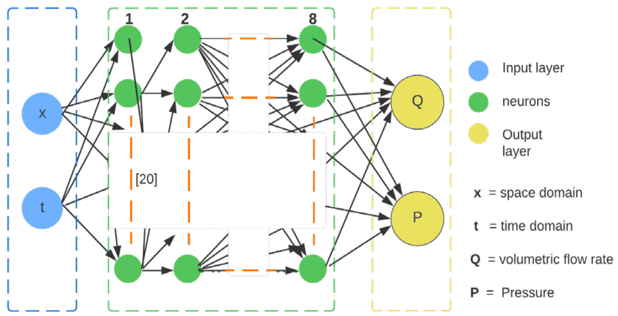


Fig. 6. Architecture of Neural Network

In this study, Scientific Artificial Neural Network (SciANN) library is adopted for shaping the architecture of neural network.
Scientific Artificial Neural Networks (SciANN) is an additional library based on artificial neural network that relies on Keras and TensorFlow packages [27]. The input layer consists of the space-time variables whereas the output layer consists of flow property i.e. volumetric flow rate (Q) which is a scalar quantity and the Pressure (P). The hidden layers consist of 20 layers with 8 neurons in each layer chosen for the optimum output results with less error. The domain for the space and time limit is selected for the required view of output and the constants used in the models are obtained from the past literature and verified data sources as in Table-1. It also includes automatic differentiation and high computational abilities, thus thriving in the present computational science.

## III. RESULTS

This study investigates about the flow rate and pressure through the carotid artery with defined geometry of plaque. For the mathematical model and the simulation conditions stated above, using the network architecture, hyper parameters, related constant values and boundary condition associated described in previous section, flow rate and velocity pictorial plots for different condition of plaque structures are obtained using matplotlib library.

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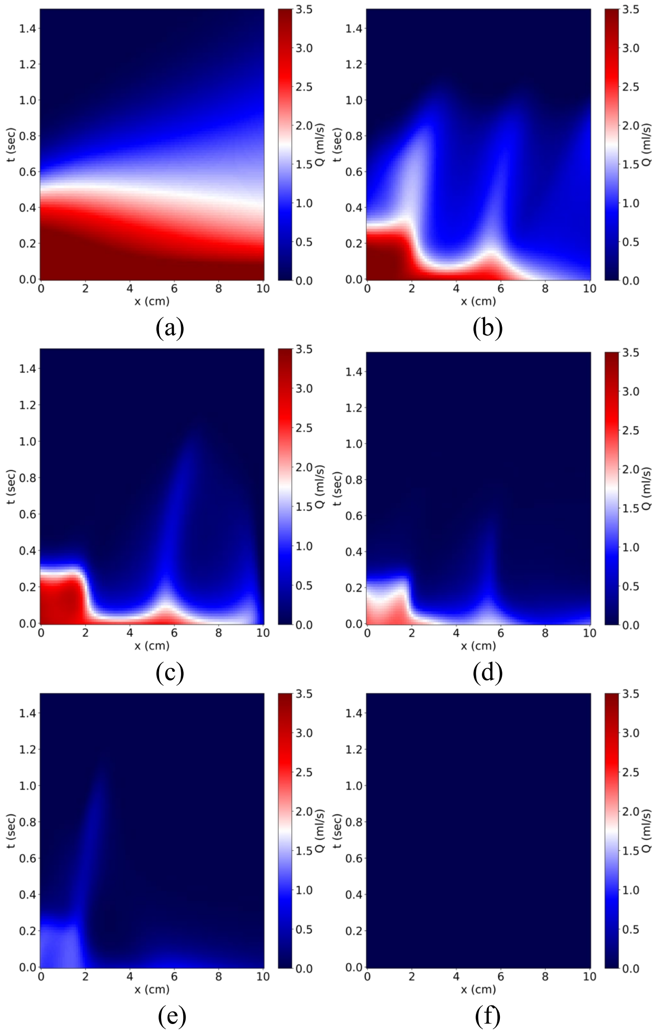


Fig. 7. Pictorial plot: Volumetric flow rate vs space and time (a) at no plaque condition (b) at 10% plaque condition (c) at 20% plaque condition (d) at 30% plaque condition (e) at 40% plaque condition (f) at 50% plaque condition

Volumetric flow rate obtained from the simulation is shown in the pictorial plots as seen in Fig 7. The x-axis consists of the space domain of the blood vessel and the y-axis represents the time domain. The values of the flow rates are plotted inside the plot with different color spots. The magnitude of the colored spots can be found in the color-bar at the right hand side of the plot. The maximum flow rate was found for the condition with no plaque (0 % plaque) and the magnitude of the flow rate is seen decreasing linearly with the increasing content of plaque. Also, the magnitude of the flow decreased in the plaque area due to the obstruction as expected. The nature of pictorial velocity plots is same as volumetric flow rate as the values for the velocity plots could be obtained. As, the area of the artery decreases in the plaque region, the velocity is expected to increase significantly with the increase in the content of plaque.

Apart from the flow rate, pressure inside the vessel is another aftermath of the model. The pressure doesn't significantly depends upon the content of plaque inside the flow but rather builds up when the flow rate or velocity decreases in magnitude due to the obstruction caused by the plaque. Although the input pressure was provided with the wave functions over time, the model indicated the identical nature of the outputs for different space points as well as for time period.

## IV. DISCUSSION

From the results obtained, different 2D plots are used for the discussion of nature of flows and its quantities through the vessel. Literatures have used traditional FEM methods to predict the flow inside the vessel conditions for different pressure conditions [4]. Also, using Doppler Ultrasound, actual values of flow rates and velocities for systolic and diastolic strokes is also presented [2]. Studies using the 1D model for different FEM, FVM techniques have presented the nature of wave propagation inside the vessel for different mechanical properties also [28], [29].

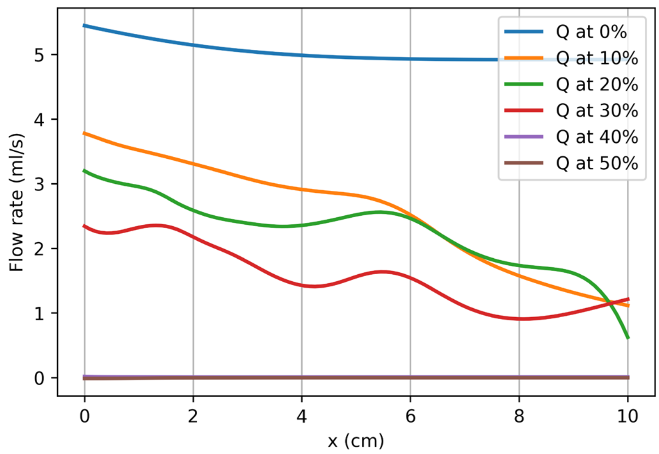


Fig. 8. Predicted flow rates for different coditions of plaque over space

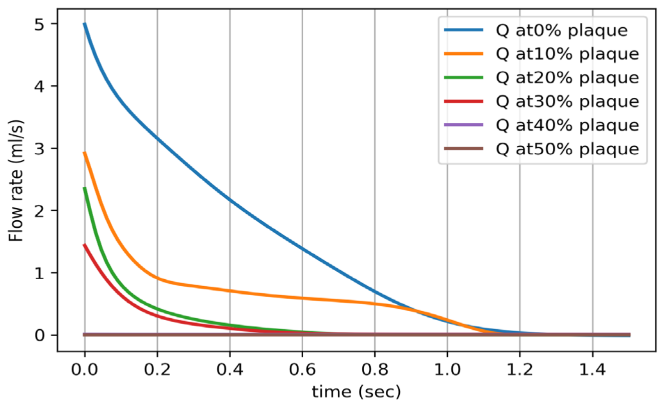


Fig. 9. Predicted flow rates for different conditions of plaque over time

The plot of flow rate for different conditions of plaque over space and time is shown in figure Fig 8, 9. For the condition with no plaque, the flow rate seems to be decreasing slightly as it passes through the vessel domain. But as the plaque is present for other conditions, the curve seems to be shaky with the decrease in magnitude. As the content of plaque in the wall increases, more fluctuations is seen on the curve. This conveys the understanding that the higher obstruction will cause the blood to reflect through the plaque and also after passing through the plaque region. For the content of plaque more than 50%, the flow rate seems to be minimum and the blood could not pass through the plaque in our simulation. But in real conditions, the blood flows through the small capillaries also. From the assumptions made in the model, the viscosity of the blood is kept constant. But, blood being the non-Newtonian fluid, changes its viscosity under different stress conditions. Considered as shear-thinning fluid, blood can drop viscosity in order to pass through extreme small conditions. Therefore, the curves after 50% plaque are same and the magnitude of the flow rate is insignificant. This result also indicates that, for flow metrics used for the study, plaque more than 50% is harmful for the health specially in the carotid artery which provides oxygen to the head and brain part of the body. But, in practical Newtonian conditions of blood, literature suggests plaque more than 70% to be medically alarming condition.

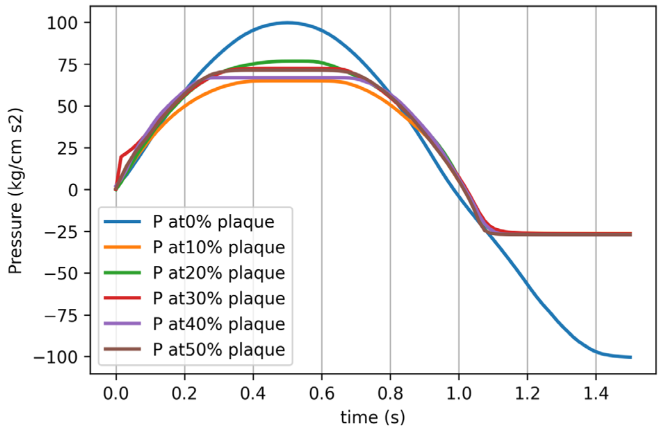


Fig. 10. Predicted pressure for different conditions of plaque over time

The pressure plots for each of the cases of plaques inside the artery is seen to be similar in terms of nature and magnitude. The data of the pressure plot leaned towards the statement that the pressure is independent of arterial geometry. In the absence of plaque, the pressure curve follows the input wave up to 0.5 seconds. After 0.5 seconds, the systolic and diastolic waves were provided as input, but the predicted pressure falls rapidly and ends into negative pressure at the end of simulation. The negative pressure represents contraction of the vessel which may be caused by the systolic and diastolic waves that causes opposite reaction to elastic deformation. The condition with no plaque provides the near equal negative pressure whereas the condition with different conditions of plaque provides one third of negative pressure as shown in figure Fig 10. This information suggests that the pressure inside the flood vessel primarily depends upon the mechanical characteristics, particularly elastic property of the nature of lipid through which blood flows.

## V. CONCLUSION

The flow simulation for different conditions of the human carotid atherosclerotic plaques with one dimensional blood flow model was done using Physics Informed Neural Network (PINN) which uses advance machine learning tools and techniques. The size of the artery for Atherosclerosis was reduced with respect to the ratio of the original artery radius. The mathematical model was provided with suitable boundary conditions and other constant parameters related to the blood. The architecture of the neural network was tuned for the best results and the results for flow rate, velocity and pressure was drawn from the model for time and space. The results obtained from different conditions of plaque in the artery suggested that the rise in the content of plaque obstructs the flow of blood in the artery increasing the pressure ahead of the obstruction. Until the reduction of radius is half (50%), the blood passes through the vessel with significant magnitude but, on further increase of plaque, there is insignificant flow of blood. This conveys that, plaque more than 50% of its radius, or 75% of its area can become fatal for human body and clicks for medical emergency. In terms of pressure, it was found that no significant change in pressure difference is simulated for different contents of plaque. The nature of pressure of the flow depend the mechanical characteristics of the vessel rather than the content of plaque on the internal walls. The study signifies the use of neural network capability and effectiveness in the mathematical flow modelling governed by partial differential equation.

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